Name: ______________________________ Date: ____________________

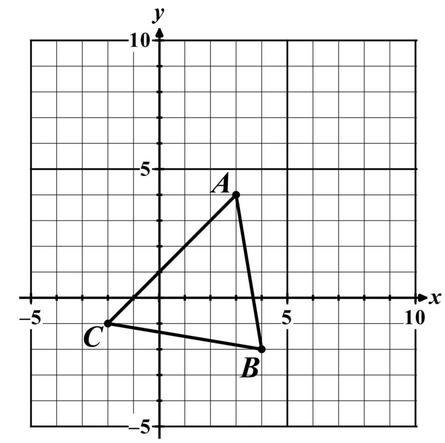
DILATIONS
N-GEN MATH® GEOMETRY

So far, we have concentrated on ***transformations*** *that* ***preserve*** *both* ***distances*** *and* ***angles****. These transformations are known as* ***rigid motions****. There is a very important type of transformation that preserves the* ***shape*** *of an object, but not its* ***size****. It is known as a* ***dilation****.*

***Exercise* #1:** ΔABC is shown plotted with vertices at $A(3,4)$, $B(4,-2)$, and $C(-2,-1)$. Given the following transformation rule:

$$f:(x,y)\rightarrow(2x,2y)$$

Map ΔABC using the rule. Label the image $\Delta A'B'C'$. Show the mapping below.



The image you created in Exercise #1 is clearly larger than its preimage but still has the same shape. We now formally define what a ***dilation*** *is.*

Dilations

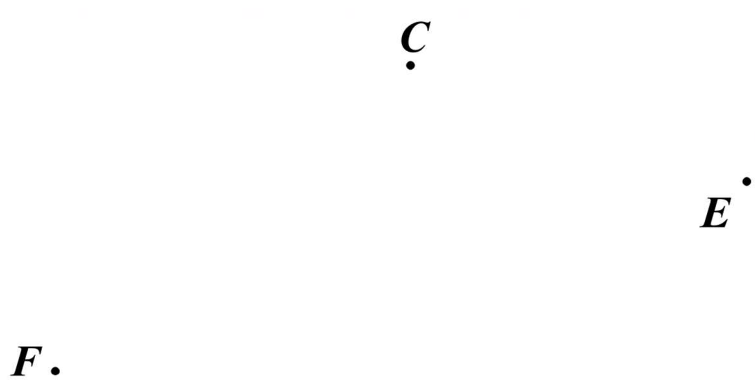
A dilation ***D*** with a **center** at **point *C*** and a **scale factor** of ***k*** maps a point ***A*** to an image point $\boldsymbol{A'}$ such that:

1. If ***A*** is the **center** of the dilation, i.e., ***C***, then $\boldsymbol{D(A)=A}$. Dilations **do not move** the **center point**.
2. If ***A*** is **not** the center of dilation then $\boldsymbol{D(A)=A'}$ where $\boldsymbol{A'}$ is located on $\overrightarrow{\boldsymbol{CA}}$ such that $\boldsymbol{CA'=k\cdot CA}$.

***Exercise* #2:** Given points *C*, *E*, and *F* below, use a ruler to plot the image of each of the following:

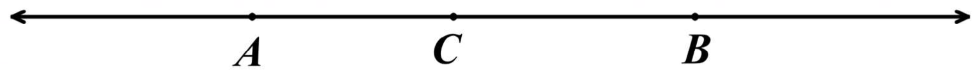
(a) a dilation of *E* with a center of *C* and a scale factor of $k=2$. Label as E'.

(b) a dilation of *F* with a center of *C* and a scale factor of $k=\frac{1}{2}$. Label as F'.



***Exercise* #3:** Given point C that lies on $\overleftrightarrow{AB}$ shown below do the following:

(a) Using a compass and straightedge, dilate both A and B using C as the center and a scale factor equal to 2.

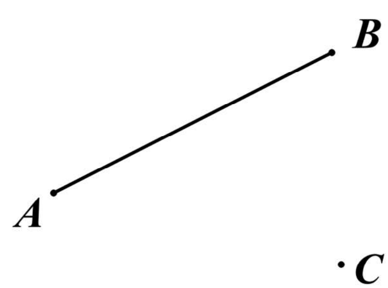


(b) What can you say about the two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{A'B'}$?

*When a **segment** is dilated using a center that does **not** lie on the line segment, two interesting things happen.*

***Exercise* #4:** Given $\overline{AB}$ shown below along with point C do the following:

(a) Using a compass and straightedge, construct the image of $\overline{AB}$ after a dilation with a center at C and a scaling factor of $k=2$. Leave all marks. Label the image $\overline{A'B'}$.



(b) Measure the lengths of $\overline{AB}$ and $\overline{A'B'}$ in centimeters. Round to the nearest tenth if needed.

$AB=$ __________ $A'B'=$ __________

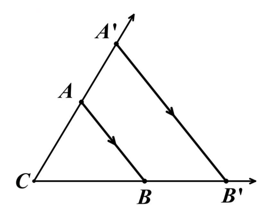
(c) What is true about the lengths of the segments? Write an equation relating them.

(d) What else appears to be true about the two segments?

Fundamental Theorem of Dilations

If a line segment, $\overline{AB}$, is dilated using a **center point** C **not** on $\overleftrightarrow{AB}$ and a **scale factor** of k then:

1. $A'B'=k\cdot AB$ and 2. $\overline{A'B'}\parallel\overline{AB}$

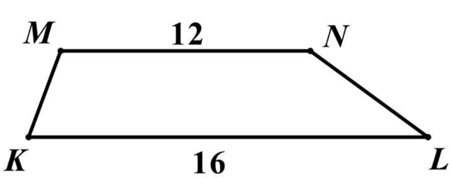


***Exercise* #5:** In the image below, $\overline{MN}$ is the image of $\overline{KL}$ after a dilation centered at point C.

(a) Locate and label point C.

(b) Determine the scaling constant, k.

(c) Explain why $MNLK$ must be a trapezoid.



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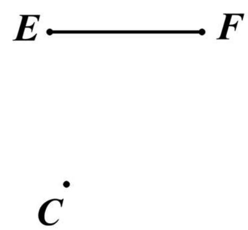
DILATIONS
N-GEN MATH® GEOMETRY HOMEWORK

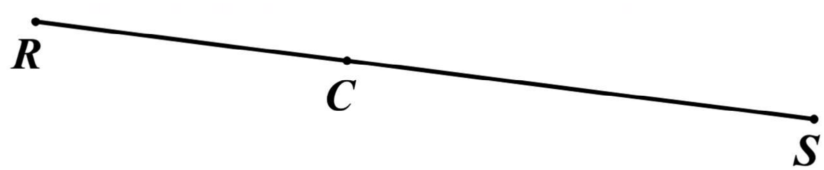
FLUENCY

1. Use a straightedge and compass only, construct the image of $\overline{EF}$ after a dilation centered at C with a scale factor of $k=3$. Label its image $\overline{E'F'}$. Leave all construction marks.

2. Verify using your compass that $E'F'=3\cdot EF$ on the diagram above. Leave your construction marks.



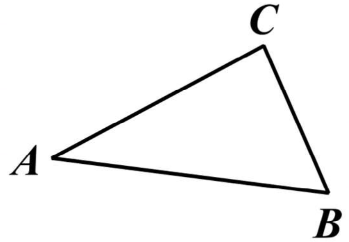
3. In the diagram below, point C lies on $\overline{RS}$. Using a ruler, draw the image of $\overline{RS}$ after a dilation with a center at C and a scale factor of $k=\frac{1}{2}$. Label the image $\overline{R'S'}$.



4. Using your ruler, measure the lengths of $\overline{RS}$ and $\overline{R'S'}$ in centimeters. Verify that $R'S'=\frac{1}{2}\cdot RS$.

$RS=$ _______________ $R'S'=$ _______________

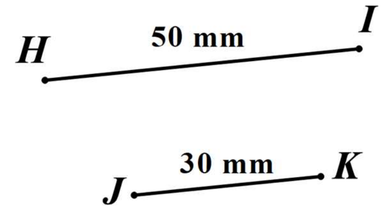
5. Using only a compass and straightedge, dilate , ΔABC shown below, using a center at point A and a scale factor of 2. Leave all construction marks. Label the image $\Delta A'B'C'$.



6. In the diagram below, $\overline{JK}$ is the image of $\overline{HI}$ after a dilation with a center at point C (not shown) and with a scale factor of k.

(a) Using only a straightedge, locate and label the center of dilation.

(b) Without calculating its value, why can you say that the scale factor has a value less than 1?



(c) What is the value of k?

7. If $\overline{LM}$ is dilated by a scale factor of 5 with a center point not on $\overline{LM}$, then which of the following would be true about its image $\overline{L'M'}$?

(1) $\overline{L'M'} \perp \overline{LM}$ and $L'M' = 5LM$

(2) $\overline{L'M'} \perp \overline{LM}$ and $L'M' = \frac{1}{5}LM$

(3) $\overline{L'M'} \parallel \overline{LM}$ and $L'M' = 5LM$

(4) $\overline{L'M'} \parallel \overline{LM}$ and $L'M' = \frac{1}{5}LM$ ________

8. If $\overline{RS}$ is dilated by a scale factor of 4 with a center of R, then which of the following is true about the segment joining point S with its image point S'? Hint: draw a good picture!

(1) $SS' = 4RS$ and $\overline{SS'} \parallel \overline{RS}$

(2) $SS' = \frac{1}{4}RS$ and $\overline{SS'}$ lies on top of $\overline{RS}$

(3) $SS' = 3RS$ and $\overline{SS'}$ lies on $\overleftrightarrow{RS}$

(4) $SS' = \frac{1}{3}RS$ and $\overline{SS'} \perp \overline{RS}$ ________

REASONING

9. In the diagram of ΔABC, points D and E are the midpoints of sides $\overline{AB}$ and $\overline{AC}$ respectively. Give a dilation that would map $\overline{DE}$ onto $\overline{BC}$. State its center point and its scale factor. Explain.

Center: Scale Factor:

Explanation: