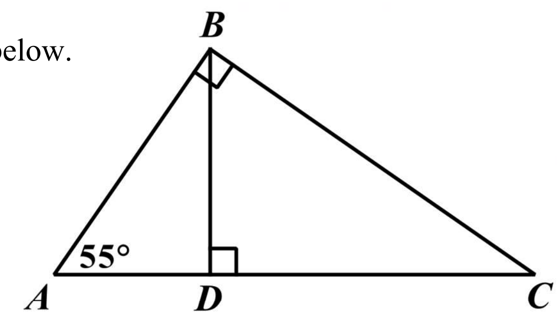
Name: ____________________________________ Date: _________________________

PARTITIONING THE HYPOTENUSE WITH AN ALTITUDE
N-GEN MATH® GEOMETRY

A very special case of ***similarity*** *arises when we draw the* ***altitude*** *from the* ***right-angle vertex*** *of a* ***right triangle*** *to its opposite side, which is the* ***hypotenuse****. This altitude partitions (divides) the hypotenuse in a very specific way due to similarity.*

***Exercise* #1:** Right triangle ABC is shown below with $\angle ABC$ as its right angle. The altitude from B to **hypotenuse** $\overline{AC}$ has been drawn. It is known that $m\angle BAD = 55°$ as marked.



(a) How many right triangles are in the diagram? Write their names below.

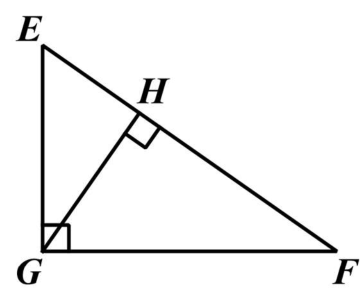
(b) Fill in the measures of all acute angles in the diagram.

(c) Why can we say that all of the right triangles from (a) are similar?

(d) Draw the two smaller right triangles in the same orientation as the larger one. Then, write a similarity statement being careful to make sure corresponding vertices are in the correct order.

In general, anytime the ***altitude*** *is drawn from the* ***right-angle vertex*** *to the* ***hypotenuse****, three* ***similar right triangles*** *are created.*

***Exercise* #2:** In the diagram below of ΔEFG, the altitude $\overline{GH}$ has been drawn to the hypotenuse $\overline{EF}$. It is known that $EH = 2$ and $HF = 8$.



(a) Mark all congruent acute angles in the diagram.

(b) Draw the two smaller right triangles below the larger one, but in the same orientation.

(c) Use similarity of the two smaller triangles to determine the length of $\overline{GH}$.

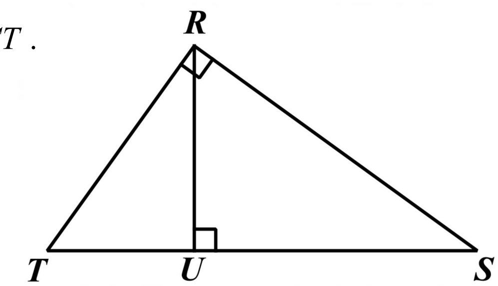
(d) Use similarity of the smaller right triangle with the original to find the length of $\overline{GE}$. Express your answer in simplest radical form.

Problems involving this type of similarity can be challenging because the three right triangles are all oriented differently. The key will always be in understanding which sides correspond with which.

***Exercise* #3:** In the diagram of right triangle *RST*, altitude $\overline{RU}$ is drawn to hypotenuse $\overline{ST}$ as shown.

(a) Mark all congruent acute angles on the diagram.

(b) Draw the two smaller right triangles in the same orientation as ΔRST. Use part (a) to help.



(c) If $RU = 10$ and $TU = 5$, then find the length of $\overline{SU}$.

Sometimes you will be asked to solve problems involving these types of triangles without being given an image to work with. Carefully draw a good diagram using everything given in the problem.

***Exercise* #4:** In ΔMNP, where $\angle P$ is a right angle, the altitude $\overline{PL}$ is drawn to hypotenuse $\overline{MN}$. If $ML = 9$ and $LN = 3$, then find the length of $\overline{PN}$.

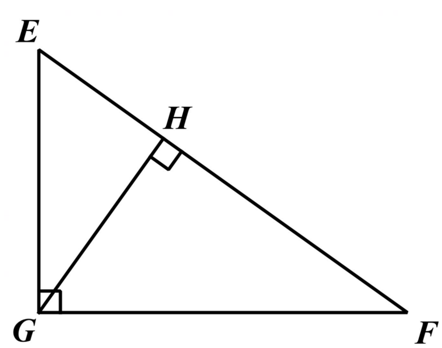
Recall that two figures will be ***similar*** *only if a* ***similarity transformation*** *exists that will map one figure onto another figure. Since these problems involve three similar right triangles, those mappings must exist.*

***Exercise* #5:** In the diagram shown below, altitude $\overline{GH}$ has been drawn from point *G* to hypotenuse $\overline{EF}$.

(a) Mark all congruent acute angles in the diagram.

(b) Draw ΔEHG on tracing paper. Use this drawing to verify that ΔEHG shares the same angles as ΔGHF.

(c) Describe a similarity transformation that would map ΔEHG onto ΔGHF. Be as specific as possible.



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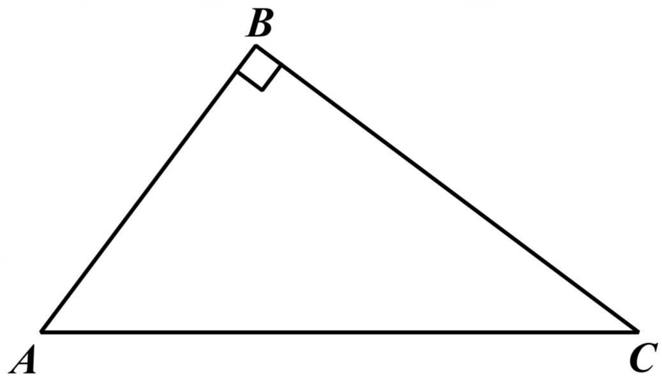
Partitioning the Hypotenuse with an Altitude
N-Gen Math® Geometry Homework

Fluency

1. For right triangle ABC shown below, using only a straightedge and compass, construct the altitude $\overline{BD}$ from B to $\overline{AC}$. Leave all construction marks. (See Unit 4 – Lesson 5 on Constructing a Perpendicular Line from a Point Not on the Line).



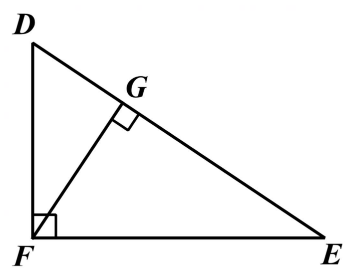
2. In the diagram below of right triangle DEF, altitude $\overline{FG}$ has been drawn from point F to $\overline{DE}$. If $DG = 4$ and $GE = 8$, then which of the following is the length of $\overline{FG}$?

(1) 6

(2) $4\sqrt{2}$

(3) $6\sqrt{2}$

(4) 12



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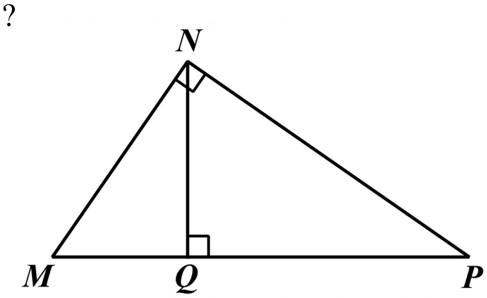
3. In the diagram below, altitude $\overline{NQ}$ has been drawn to hypotenuse $\overline{MP}$ in right triangle MNP. Which of the following would be equal to the ratio of MN to NQ?

(1) the ratio of NP to PQ

(2) the ratio of NQ to NP

(3) the ratio of NP to MN

(4) the ratio of NP to PM



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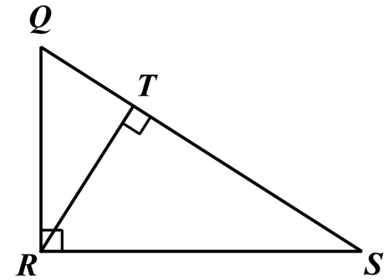
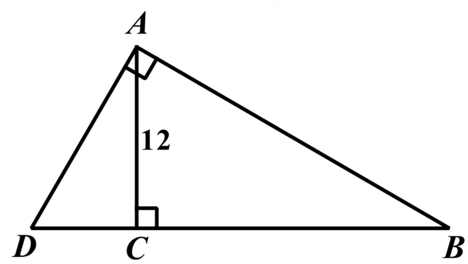
4. In the diagram shown of right triangle QRS, altitude $\overline{RT}$ is drawn to $\overline{QS}$. If $RS = 20$ and $QS = 25$, then find the length of $\overline{ST}$. Show how you arrived at your answer.



5. The altitude drawn to the hypotenuse of right triangle EFG partitions the hypotenuse into two segments of lengths 8 and 10. Find the length of the longer leg of ΔEFG in simplest radical form.

6. In the diagram below, altitude $\overline{AC}$ is drawn from a right angle to $\overline{BD}$ in such a way that $BC : DC = 4 : 1$. If $AC = 12$, then find the length of $\overline{DC}$ algebraically. Show your work.

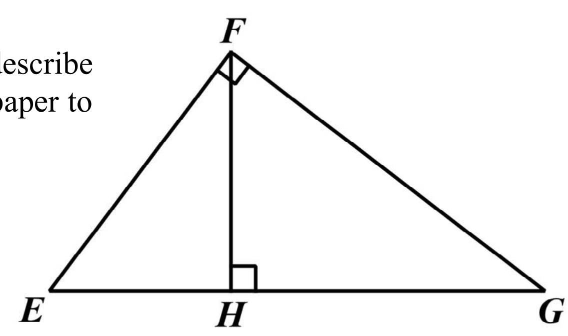


Reasoning

7. In the diagram of right triangle EFG shown, altitude $\overline{FH}$ has been drawn from point F to hypotenuse $\overline{EG}$.

(a) Mark all congruent acute angles in the diagram.

(b) If ΔEHF was rotated by $90°$ clockwise about point H, describe where image points E' and F' would land. Use tracing paper to draw a sketch of $\Delta E'H'F'$.



(c) What would be true about $\overline{E'F'}$ and $\overline{FG}$? Justify.