Name: ______________________________________ Date: __________________________

GEOMETRIC MEANS
N-GEN MATH® GEOMETRY

When we calculate the ***arithmetic mean*** *of* ***two numbers****, we do so by* ***adding*** *the two numbers together and dividing by 2. This results in a number that is* ***half-way*** *between the two being averaged. There are other types of means, though. One of which arises in geometry, appropriately named the* ***geometric mean****.*

The Geometric Mean of Two Numbers

If m and n are two **positive numbers**, then their **geometric mean** is given by $\sqrt{m \cdot n}$.

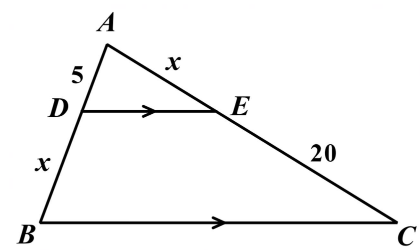
***Exercise* #1:** Find the geometric mean of each set of numbers given. If the answer is *not* an integer, then write the geometric mean in simplest radical form.

(a) 2 and 8 (b) 4 and 9 (c) 5 and 10 (d) 6 and 9

Geometric means get their name from geometry because they arise in many contexts.

***Exercise* #2:** What is the side length of a square that has the same area as a rectangle with a width of 9 feet and a length of 16 feet? Show how you arrived at your answer. Draw diagrams to support your work.

***Exercise* #3:** In the diagram below, points D and E lie on $\overline{AB}$ and $\overline{AC}$ such that $\overline{DE} \parallel \overline{BC}$. If $AD = 5$, $EC = 20$, and $AE = DB$:



(a) Find the length of $\overline{AE}$.

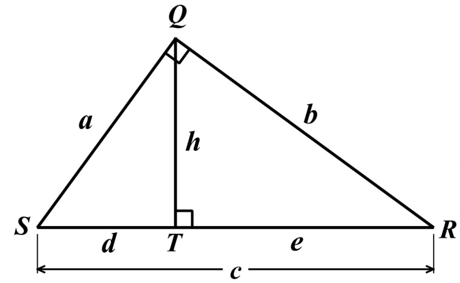
(b) Does your answer to (a) make sense in terms of the Side Splitter Theorem? Explain.

The Geometric Mean of Two Numbers as a Solution

The **geometric mean** of numbers m and n is the positive solution to the equation $\frac{m}{x} = \frac{x}{n}$.

Geometric means arise in our problems regarding the ***altitude drawn to the hypotenuse****.*

***Exercise* #4:** Given right triangle QRS shown below with altitude $\overline{QT}$ partitioning hypotenuse $\overline{RS}$ into segments with the lengths given:



(a) Sketch the two smaller right triangles below in the same orientation.

(b) Using the similarity of the two triangles from (a), show that the length of the altitude, h, is the geometric mean of the lengths of the two partitioned segments of the hypotenuse $\overline{RS}$, d and e.

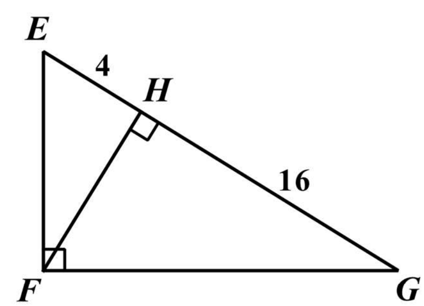
(c) Draw the smaller right triangle in the same orientation as the original right triangle.

(d) Show that the length of $\overline{QS}$ is the geometric mean of the length of $\overline{SR}$ and the length of $\overline{ST}$.

Geometric Means and Right Triangles

1. The **altitude** to the **hypotenuse** of a right triangle has a length that is the **geometric mean** of the **two segment lengths** partitioned along the hypotenuse (most common).
2. The **length** of **either leg** of the original right triangle is the **geometric mean** of the **length** of the **hypotenuse** with the **length** of the **partitioned segment** of the **hypotenuse adjacent** to the **leg**.

***Exercise* #5:** Given right triangle EFG, altitude $\overline{FH}$ is drawn to hypotenuse $\overline{EG}$, partitioning it into lengths of 4 and 16.



(a) Determine the length of altitude $\overline{FH}$.

(b) Determine the length of side $\overline{EF}$ in simplest radical form.

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GEOMETRIC MEANS
N-GEN MATH® GEOMETRY HOMEWORK

FLUENCY

1. Find the geometric mean of each set of numbers. If they are not integers, express your answers in simplest radical form.

(a) 5 and 20 (b) 8 and 18 (c) 5 and 15 (d) $\frac{1}{2}$ and 40

2. The geometric mean of the positive numbers a and b is the positive solution to which of the following equations?

(1) $ax = b$ (3) $x^2 = \frac{b}{a}$

(2) $\frac{a}{x} = \frac{x}{b}$ (4) $\frac{x}{a} = \frac{x}{b}$

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3. A rectangle has a length of 50 feet and a width of 8 feet. How long would be one side of a square that has the same area as this rectangle?

(1) 15 feet

(2) 18 feet

(3) 20 feet

(4) 29 feet

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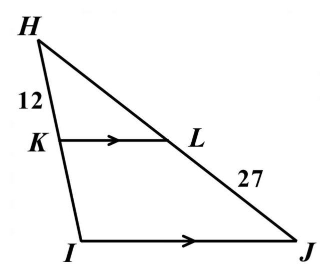
4. In the diagram shown, points K and L lie on $\overline{HI}$ and $\overline{HJ}$ such that $\overline{KL} \parallel \overline{IJ}$ and $HL = KI$. Which of the following is the ratio of HK to KI?



(1) 2 to 3

(2) 3 to 4

(3) 3 to 5

(4) 4 to 9

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5. An altitude drawn from the right angle of a right triangle partitions the right triangle's hypotenuse into segments of lengths 8 inches and 5 inches. Which of the following is closest to the length of the altitude?

(1) 5.98

(2) 6.32

(3) 6.71

(4) 7.22

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6. The altitude drawn from the right angle of a right triangle partitions its hypotenuse into lengths of 18 and 14. Which of the following is the length of the right triangle's longer leg?

(1) 24

(2) 32

(3) $6\sqrt{7}$

(4) $8\sqrt{3}$

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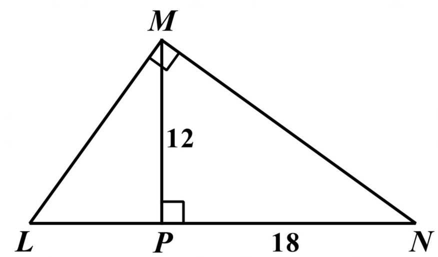
7. In the diagram shown below of right triangle *LMN*, the altitude from *M* to $\overline{LN}$ has been drawn. Given that $MP = 12$ and $PN = 18$:



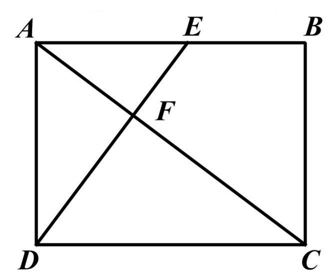
(a) Set up and solve a proportion that solves for LP.

(b) Use a geometric mean and your answer to (a) to find the length of $\overline{LM}$ in simplest radical form.

(c) Instead of using a geometric mean, find the length of $\overline{LM}$ in simplest radical form using your answer from (a) and the Pythagorean Theorem. Compare your answer to (b).

8. In the image shown, *ABCD* is a rectangle. Point *E* is located on $\overline{AB}$ such that $\overline{DE}$ is perpendicular to $\overline{AC}$. Diagonal $\overline{AC}$ is partitioned by $\overline{DE}$ into segments with lengths $AF = 16$ and $FC = 36$.

Determine the length of $\overline{FE}$. Show the work that leads to your answer. (Warning: Answer will not be a whole number.)



Reasoning

9. An altitude to the hypotenuse partitions it in such a way that the longer segment is four times greater in length than the shorter segment. Show that the length of the altitude must be twice the length of the shorter segment.