



## DILATIONS AND ANGLES N-GEN MATH® GEOMETRY



We have now seen in both the Euclidean and coordinate geometry planes the various properties of dilations.

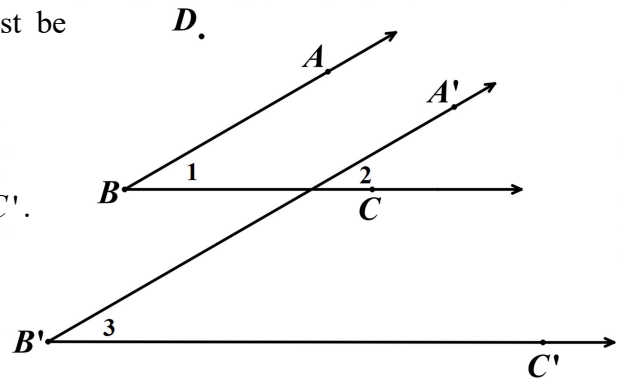
### Properties of Dilations

1. A dilation **maps a line not** passing through the dilation's **center** to another **parallel line**.
2. A dilation **maps a line passing** through the dilation's **center** to the same line.
3. A dilation **maps a line segment** to another line segment whose **length** has been **scaled** by the same scaling factor as the scale constant for the dilation.

Dilations also have another important property that is a direct consequence of the first property above.

**Exercise #1:** In the diagram shown,  $\triangle ABC$  has been dilated with a center at  $D$  and a scale factor of 2.

- (a) Based on the first property of dilations, list rays that must be parallel in this diagram.



- (b) Use a protractor to find the measures of  $\angle ABC$  and  $\angle A'B'C'$ . What is true of these two measures?

$m\angle ABC =$  \_\_\_\_\_       $m\angle A'B'C' =$  \_\_\_\_\_

- (c) Give an explanation based on (a) for why  $\angle 1$  must be congruent to  $\angle 3$ .

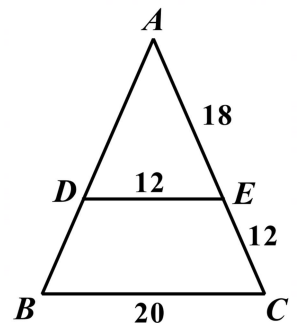
### An Additional Property of Dilations

**Dilations preserve the measure of angles.** When a geometric figure is dilated, **its size will change** but its **angles will remain the same**. This means that the image and preimage will have the **same shape**.

**Exercise #2:** In the image,  $\triangle ADE$  is the image of isosceles  $\triangle ABC$  after a dilation with a center at point  $A$ .

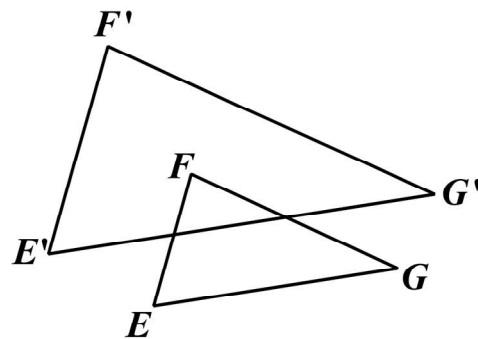
- (a) What is the scale factor for this dilation?

- (b) If  $m\angle A = 46^\circ$ , then what is the measure of  $\angle ADE$ ? Justify.



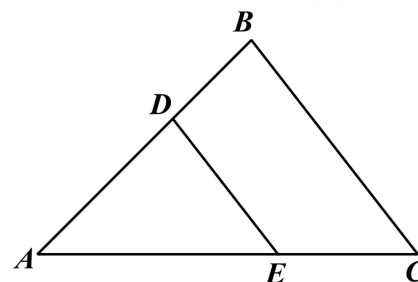
**Exercise #3:** In the diagram shown,  $\triangle E'F'G'$  is the image of  $\triangle EFG$  after a dilation with an unknown center and an unknown scale factor.

- Use tracing paper to verify that the angle measures have been preserved.
- Find the center of dilation using a straightedge. Mark it as point  $D$ . Leave all construction marks.
- By measuring corresponding sides, determine the scale factor to the nearest tenth. Do at least two calculations.



**Exercise #4:** In the diagram shown, points  $D$  and  $E$  have been located on  $\overline{AB}$  and  $\overline{AC}$  such that  $\overline{DE} \parallel \overline{BC}$ .

- Give a dilation that will map  $\overline{BC}$  onto  $\overline{DE}$ . Specify both the center and the scale factor of the dilation. Explain your choices.



- If  $\triangle ABC$  was transformed using the dilation from (a), explain why  $\triangle A'B'C'$  would be congruent to  $\triangle ADE$ .

- If  $AD = 10$ ,  $AB = 15$ ,  $AE = 14$ , and  $BC = 12$ , algebraically determine the lengths of  $\overline{DE}$  and  $\overline{AC}$ .



Name: \_\_\_\_\_

Date: \_\_\_\_\_

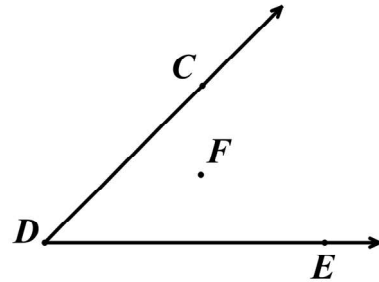


## DILATIONS AND ANGLES

### N-GEN MATH® GEOMETRY HOMEWORK

#### FLUENCY

1. Given  $\angle CDE$  shown, construct its image,  $\angle C'D'E'$ , after a dilation using point  $F$  as the center and a scale factor of 2. Leave all construction marks.



2. Using your protractor, find the measures of both angles:

(a)  $m\angle CDE =$  \_\_\_\_\_

(b)  $m\angle C'D'E' =$  \_\_\_\_\_

3. Why isn't the measure of  $\angle C'D'E'$  twice the measure of  $\angle CDE$ ?

4. Which of the following is *not* a property of dilations?

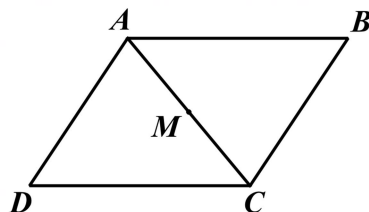
- (1) they map a line not containing the dilation's center to a parallel line
  - (2) they preserve the measures of angles between the preimage and image
  - (3) they preserve the lengths of segments between the preimage and image
  - (4) they map a line containing the dilation's center to the same line
- \_\_\_\_\_

5. If rectangle  $RSTU$  is dilated with a scale factor of  $\frac{1}{2}$  with a center at point  $R$ , then which of the following would be the value of  $m\angle S'T'U'$  in its image  $R'S'T'U'$ ?

- (1)  $30^\circ$
  - (2)  $45^\circ$
  - (3)  $90^\circ$
  - (4)  $180^\circ$
- \_\_\_\_\_

6. In parallelogram  $ABCD$  shown,  $m\angle DAB = 124^\circ$  and  $M$  is the midpoint of diagonal  $\overline{AC}$ . If  $ABCD$  is dilated using  $M$  as the center with a scale factor of 1.5, then which of the following would be the measure of  $\angle A'B'C'$  in its image  $A'B'C'D'$ ?

- (1)  $56^\circ$
  - (2)  $84^\circ$
  - (3)  $122^\circ$
  - (4)  $186^\circ$
- \_\_\_\_\_

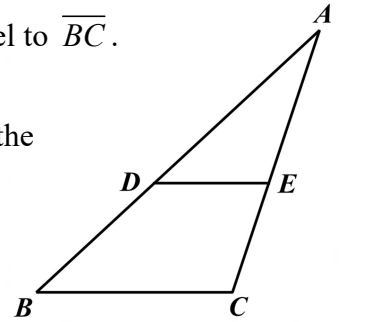


7. In isosceles triangle  $ABC$ , it is known that  $\overline{AB} \cong \overline{AC}$ . If  $\triangle ABC$  is dilated using point  $B$  as the center, then which of the following could be an incorrect statement? (Draw a good diagram!)

- (1)  $\angle BAC \cong \angle BA'C'$
- (2)  $\angle C'A'B \cong \angle ABC$
- (3)  $\angle ABC \cong \angle A'C'B$
- (4)  $\angle BCA \cong \angle A'BC'$

8. In the following diagram,  $D$  and  $E$  lie on  $\overline{AB}$  and  $\overline{AC}$  such that  $\overline{DE}$  is parallel to  $\overline{BC}$ . It is known that  $AD = 18$  and  $DB = 12$ .

- (a) Describe a dilation that would map  $\overline{BC}$  onto  $\overline{DE}$ . State both the center of the dilation and the scale factor.



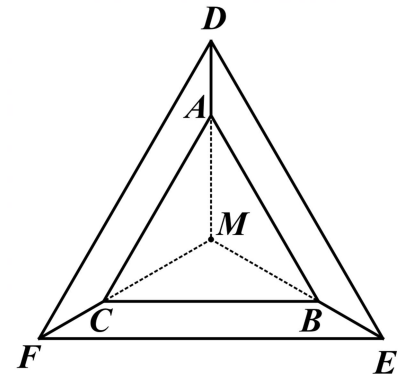
- (b) If  $BC = 15$  and  $AC = 20$ , the find the perimeter of  $\triangle ADE$ . Show how you arrived at your answer.

**REASONING**

9. In the diagram below,  $\triangle ABC$  is equilateral with a **circumcenter** at point  $M$ .  $\triangle DEF$  is the image of  $\triangle ABC$  after a dilation with a center at  $M$ .

- (a) Is the scale factor of this dilation less than or greater than 1? Explain.

- (b) Why would  $\overline{AD}$ ,  $\overline{BE}$ , and  $\overline{CF}$  all be the same length?



- (c) Explain why quadrilateral  $ACFD$  must be an isosceles trapezoid.

