Name: ___________________________________ Date: _________________________

SIMILARITY CRITERIA
N-GEN MATH® GEOMETRY

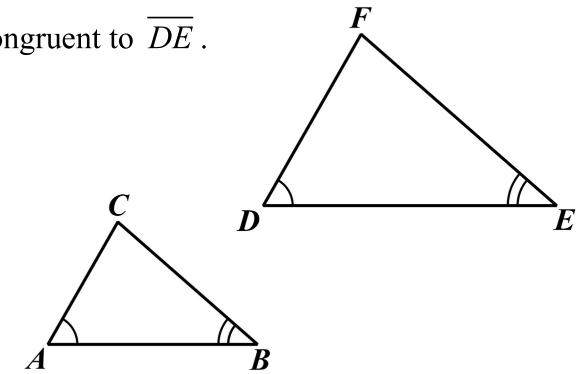
*We now know that two **similar figures** will have **congruent corresponding angles** and **corresponding sides** that are **proportional** (one set of lengths is a **constant multiple** of the other). Like **congruent triangles**, there are ways we can tell that **two triangles** are **similar** with just a **limited amount of information** (known as **criteria**). The key to understanding these criteria is the following:*

Similar Figures

Two geometric figures are **similar** if a **dilation** of **one figure** will make it **congruent** to the **other figure**.

***Exercise* #1:** Given ΔABC and ΔDEF shown below. It is **only known** that $\angle A \cong \angle D$ and $\angle B \cong \angle E$ as marked.

(a) Give a dilation of ΔABC that would make the image of $\overline{AB}$ congruent to $\overline{DE}$. Note that it does not need to map onto $\overline{DE}$, just be congruent.



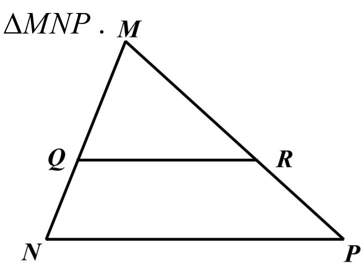
(b) If ΔABC was dilated using the dilation from (a), would $\angle A' \cong \angle A$ and $\angle B' \cong \angle B$?

(c) Explain why $\Delta A'B'C'$ from the dilation in (b) would be congruent to ΔDEF.

The Angle-Angle Criterion for Similarity (AA)

If **two angles** of one triangle are **congruent** to **two angles** of another triangle, then the **triangles are similar**.

***Exercise* #2:** In the diagram shown, points Q and R lie on $\overline{MN}$ and $\overline{MP}$ such that $\overline{QR} \parallel \overline{NP}$. Explain how the Angle-Angle Criterion for Similarity allows you to conclude that ΔMQR is similar to ΔMNP.



***Exercise* #3:** If, in the diagram, $MQ = 8$, $NQ = 6$, and $NP = 21$, determine the length of $\overline{QR}$.

There are two additional sets of criteria that can be used to ***prove two triangles*** *are* ***similar****. We will not formally prove these but will demonstrate why they must be true. First, the two sets of criteria.*

The Side-Side-Side Criterion for Similarity (SSS)

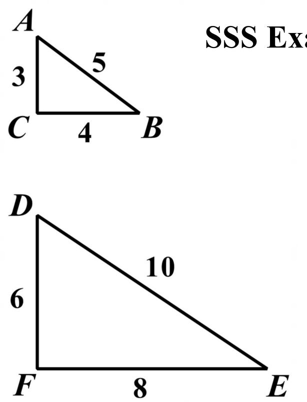
If **all three pairs** of **corresponding sides** of the two triangles are **proportional**, then the triangles are **similar**.

The Side-Angle-Side Criterion for Similarity (SAS)

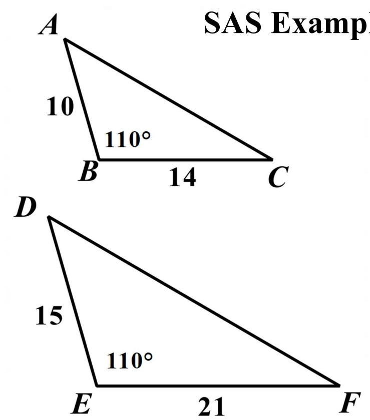
If **two sides** of one triangle are **proportional** to **two sides** of another triangle and their **included angles** are **congruent**, then the two triangles are **similar**.

***Exercise* #4:** In the following two diagrams, only the information marked is what is known. In both cases, give a dilation of ΔABC that will cause its image $\Delta A'B'C'$ to be congruent with ΔDEF. Justify the congruence of $\Delta A'B'C'$ and ΔDEF using a **triangle congruence theorem**.

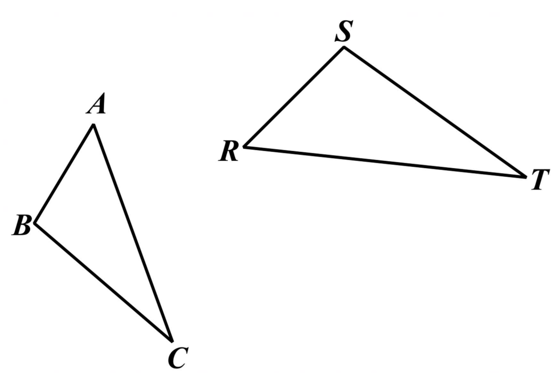
(a) **SSS Example**



(b) **SAS Example**



***Exercise* #5:** In the diagram below, which of the following sets of information would *not* be sufficient to prove that ΔABC is similar to ΔRST ?



(1) $\angle S \cong \angle B$ and $\angle C \cong \angle T$

(2) $\dfrac{AB}{RS} = \dfrac{AC}{RT}$ and $\angle A \cong \angle R$

(3) $\dfrac{RS}{AB} = \dfrac{ST}{BC} = \dfrac{TR}{CA}$

(4) $\dfrac{RT}{AC} = \dfrac{TS}{CB}$ and $\angle B \cong \angle S$

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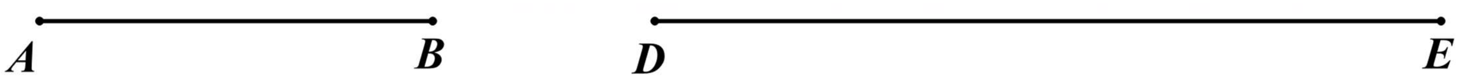
SIMILARITY CRITERIA
N-GEN MATH® GEOMETRY HOMEWORK

FLUENCY

1. In the diagram below, $\overline{DE}$ is twice the length of $\overline{AB}$. Using your protractor, create two triangles, ΔABC and ΔDEF, from these bases such that $m\angle A = m\angle D = 30°$ and $m\angle B = m\angle E = 60°$.



2. In the above diagram, measure $\overline{AC}$, $\overline{BC}$, $\overline{DF}$, and $\overline{EF}$ to the nearest millimeter. Then, calculate the ratios asked for.

$AC =$ $\qquad$ $DF =$ $\qquad$ $\frac{DF}{AC} =$

$BC =$ $\qquad$ $EF =$ $\qquad$ $\frac{EF}{BC} =$

3. The ratios you calculated in #2 above should have been equal (or close to it based on rounding errors). Why should they be equal and why are they equal to the value that they are?

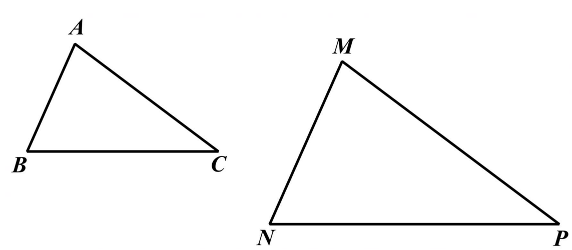
4. Which of the following would be enough information to be able to conclude that $\Delta ABC \sim \Delta MNP$?

(1) $m\angle P = m\angle C$ and $\frac{BC}{NP} = \frac{MP}{AC}$

(2) $m\angle P = m\angle C$ and $\frac{BC}{NP} = \frac{AB}{MN}$

(3) $m\angle A = m\angle M$ and $\frac{AB}{MN} = \frac{AC}{MP}$

(4) $m\angle B = m\angle N$ and $AB = MN$



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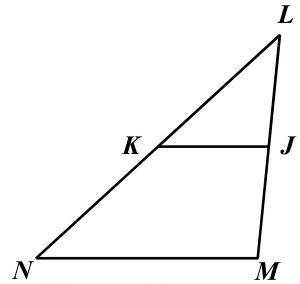
5. Given that points K and J lies on sides $\overline{LN}$ and $\overline{LM}$, which of the following measurements would justify that $\Delta LJK \sim \Delta LMN$?



(1) $KL = 8$, $LN = 12$, $LJ = 6$, and $LM = 9$

(2) $KJ = 3$, $NM = 9$, $LJ = 4$, and $JM = 8$

(3) $KJ = 8$, $NM = 16$, $KL = 11$, and $LN = 22$

(4) $KL = 12$, $LN = 20$, $LJ = 9$, and $JM = 15$

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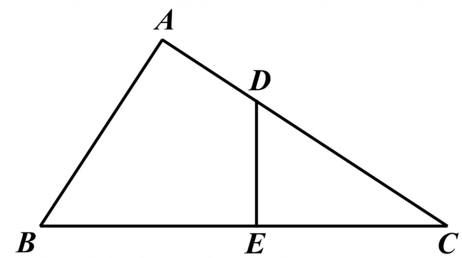
REASONING

6. In the following diagram, points D and E lie on $\overline{AC}$ and $\overline{BC}$ such that $\overline{AB} \perp \overline{AC}$ and $\overline{DE} \perp \overline{BC}$.

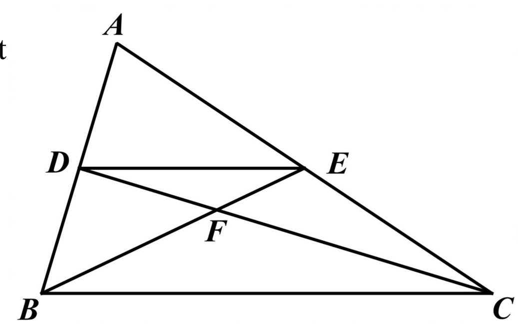
(a) Explain why ΔBAC must be similar to ΔDEC.



(b) If $BC = 20$, $DE = 4$, and $DC = 8$, then find the length of $\overline{AB}$. Show your work.

7. In the diagram of ΔABC, the midpoints of $\overline{AB}$ and $\overline{AC}$ are marked as D and E respectively. $\overline{BE}$ and $\overline{CD}$ have been drawn as shown and intersect at point F.

(a) Because D and E are the midpoints of sides $\overline{AB}$ and $\overline{AC}$, what can you conclude about $\overline{DE}$ and $\overline{BC}$? (See Unit 6, Lesson 5)



(b) What similarity criteria could be used to prove ΔDEF is similar to ΔBCF? Explain.

(c) Explain why $\overline{BF}$ must be twice as long as $\overline{FE}$.