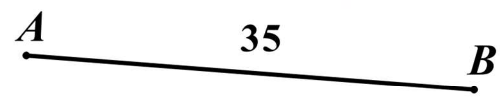
Name: ______________________________________ Date: ___________________________

PARTITIONING A LINE SEGMENT
N-GEN MATH® GEOMETRY

In the last lesson, we learned about the ***Side Splitter Theorem****. One of the applications of this theorem allows us to* ***partition*** *(or divide) a* ***line segment*** *so that its two parts are in a* ***specific ratio****.*

***Exercise* #1:** A segment is shown below whose length is 35. We would like to locate a point P on $\overline{AB}$ such that it **partitions** $\overline{AB}$ into the ratio $AP:PB=2:3$.



(a) What must be $AP:AB$? Explain.

(b) What does the ratio in (a) mean in terms of the lengths of $\overline{AP}$ and $\overline{AB}$?

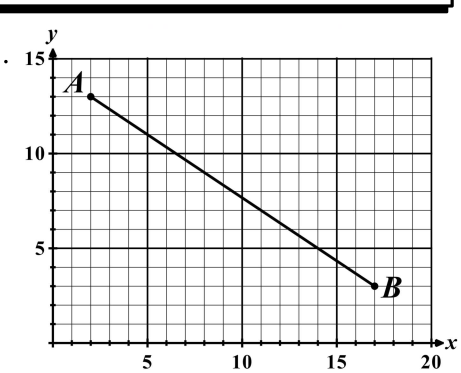
(c) Use (b) to find the length of $\overline{AP}$ using a single product (not using algebra).

We would like to be able to ***partition*** *a line segment in the* ***coordinate plane****. Before we do so, we first must introduce a new idea, the* ***directed line segment****.*

A Directed Line Segment

A directed line segment is a **typical segment** that has a **designated starting point** and **ending point**. They are useful when thinking about **motion** along a **straight-line segment**.

***Exercise* #2:** A directed line segment starts at $A(2,13)$ and ends at $B(17,3)$.



(a) Calculate the change in x, Δx, and the change in y, Δy, for this directed line segment.

(b) Illustrate these changes on the graph by drawing in a right triangle with $\overline{AB}$ as its hypotenuse and with vertical and horizontal legs.

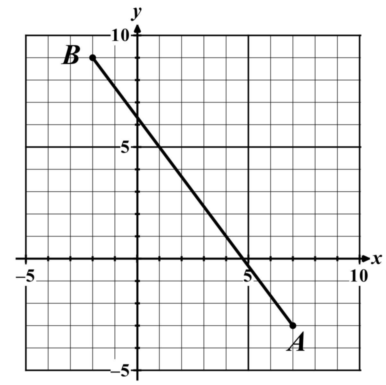
(c) A point P lies on the directed line segment that partitions it into a ratio of 2:3 (as in Exercise #1). According to the Side Splitter Theorem, how much of the change in x and change in y should we add to the starting coordinates to find the coordinates of point P? Find those coordinates of P and plot it on $\overline{AB}$.

Partitioning a Directed Line Segment in the Coordinate Plane

If a **directed line segment** that starts at $\boldsymbol{A(x_1, y_1)}$ and ends at $\boldsymbol{B(x_2, y_2)}$ is partitioned by a point P into a ratio of $\boldsymbol{m{:}n}$, then the coordinates of $\boldsymbol{P(x_p, y_p)}$ can be found using:

$$x_p = x_1 + \frac{m}{m+n}\cdot \Delta x \text{ and } y_p = y_1 + \frac{m}{m+n}\cdot \Delta y$$

***Exercise* #3:** A directed line segment starts at $A(7,-3)$ and ends at $B(-2,9)$. Point P partitions $\overline{AB}$ into a ratio of 1:2.



(a) Calculate Δx and Δy for this directed line segment.

(b) What fraction of the changes in (a) should be added to the starting coordinates to find point P? Use this to find the coordinates of P.

(c) Plot point P on $\overline{AB}$.

When we ***partition*** *a line segment, we simply* ***add*** *the* ***fraction*** *of the total line segment that goes from the* ***starting point*** *to the* ***partition point*** *to the location of the starting point.*

***Exercise* #4:** A directed line segment starts at $M(4,12)$ and ends at $N(20,-12)$. Find the coordinates of point P such that $MP:PN = 5:3$.

Of course, there is no reason why the coordinates of our partition point must turn out "nice."

***Exercise* #5:** A directed segment starts at $K(3,5)$ and ends at $L(-8,14)$. Which of the following are the coordinates of the point that partitions the directed segment into a ratio of 3 to 2?

(1) $(-4.5, 9.6)$

(2) $(-3.6, 10.4)$

(3) $(-3.2, 11.2)$

(4) $(-3.6, 9.6)$

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PARTITIONING A LINE SEGMENT
N-GEN MATH® GEOMETRY HOMEWORK

FLUENCY

1. A point *P* lies on $\overline{BC}$ such that $BP:PC=3:8$. Which of the following is an *incorrect* statement?

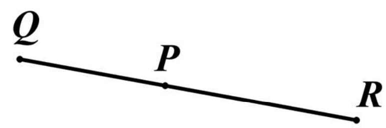
(1) $\frac{BP}{BC}=\frac{3}{11}$ (3) $\frac{PC}{BC}=\frac{8}{11}$

(2) $\frac{BC}{PC}=\frac{8}{5}$ (4) $\frac{BC}{BP}=\frac{11}{3}$ ________

2. In the diagram below, point *P* lies on $\overline{QR}$ such that $QP:PR=3:4$. Which equation below would allow you to calculate the length of $\overline{QP}$ if you know the length of $\overline{QR}$?

(1) $QP=\frac{3}{4}\cdot QR$ (3) $QP=\frac{3}{7}\cdot QR$

(2) $QP=\frac{4}{3}\cdot QR$ (4) $QP=\frac{4}{7}\cdot QR$



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3. Point *I* lies on $\overline{HJ}$ such that $HI:IJ=3:2$. If $HJ=40$, then which of the following is the length of IJ?

(1) 16

(2) 20

(3) 24

(4) 30 ________

4. A directed line segment starts at $A(-2,3)$ and ends at $B(7,18)$. A point *P* partitions the segment into a ratio of 2:1. Which of the following are the coordinates of point *P*?

(1) $(5,11)$

(2) $(6,10)$

(3) $(3,12)$

(4) $(4,13)$ ________

5. A directed line segment starts at $E(-2,12)$ and ends at $F(10,4)$. If a point *P* partitions the segment into a ratio of 1 to 3, then which of the following is the *y*-coordinate of *P*?

(1) 14

(2) 10

(3) 6

(4) 4

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6. A directed line segment starts at $A(10,5)$ and ends at $B(-11,40)$. A point *P* partitions $\overline{AB}$ into the ratio $AP:PB=3:4$. Determine the coordinates of point *P*. Show the work that leads to your answer.

7. A directed segment starts at $M(-12,-5)$ and ends at $N(8,10)$. A third point, *Q*, is located on $\overline{MN}$ such that it partitions it into a ratio of 2:3.

(a) Find the coordinates of point *Q*. Show the work that leads to your answer.

(b) Verify that $MQ:QN=2:3$ by finding the lengths of $\overline{MQ}$ and $\overline{QN}$ using the distance formula.

Length of $\overline{MQ}$: Length of $\overline{QN}$: Ratio:

APPLICATIONS

8. Using the coordinate plane, Sonia is modeling a straight-line path that she took. She started at the point $S(16,-12)$ and ended at the point $E(4,18)$. She paused along $\overline{SE}$ at point *P* when she had only half the distance left that she had already traveled from *S* to *P*. Determine the coordinates of point *P*.

REASONING

9. Consider the directed line segment from $A(-2,3)$ to $B(8,19)$. A point *P* partitions $\overline{AB}$ into a 1:1 ratio.

(a) What special point along $\overline{AB}$ is point *P*?

(b) Determine the coordinates of *P* using our partition method.

(c) Determine the coordinates of *P* using the formula for (a).